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On power corrections to the event shape distributions in QCD

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Abstract: We study power corrections to the differential thrust, heavy jet mass and C-parameter distributions in the two-jet kinematical region. We argue that away from the end-point region, $e \gg \Lambda_{\rm QCD}/Q$, the leading $1/Q$-power corrections are parameterized by a single non-perturbative scale while for $e \sim \Lambda_{\rm QCD}/Q$ one encounters a novel regime in which power corrections of the form $1/(Qe)^n$ have to be taken into account for arbitrary n. These non-perturbative corrections can be resummed and factor out into a universal non-perturbative distribution, the shape function, and the differential event shape distributions are given by convolution of the shape function with perturbative cross-sections. Choosing a simple ansatz for the shape function we demonstrate a good agreement of the obtained QCD predictions for the distributions and their lowest moments with the existing data over a wide energy interval.

Keywords: Nonperturbative Effects, LEP HERA and SLC Physics, QCD, Jets.

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Contents

1. Introduction

Analysis of hadronization effects to the final states in e^+e^--annihilation has became the subject of active QCD studies [1]. There exist infrared and collinear safe event shape variables for which perturbative QCD can be applied at large center-of-mass energies $s = Q^2$ to calculate their differential distributions and mean values as series in $\alpha_s(Q)$. It has been observed many years ago [2] that for some shape variables like thrust, t, and heavy jet mass, ρ, perturbative QCD predictions deviate from the data by corrections suppressed by powers of the large energy scale $1/Q^p$, with the exponent p depending on the variable and $p = 1$ for t- and ρ-variables. Such hadronization corrections were measured experimentally [1] over a wide energy interval $14 \leq \sqrt{s}/\mathrm{GeV} \leq 189$ and were found to have a different form for the differential event shape distributions, $d\sigma/de$, as compared to their mean values, $\langle e\rangle = \sigma_{\rm tot}^{-1}\int de\, e d\sigma/de$. For the mean value $\langle e\rangle$ the leading power correction is parameterized by a non-perturbative *scale* λ_p of dimension p, while hadronization corrections to the differential distribution are described by a *function* $f_{\rm hadr}(Q,e)$ depending on both the shape variable and the center-of-mass energy

$$\langle e\rangle = \langle e\rangle_{\rm PT} + \frac{\lambda_p}{Q^p}\,, \qquad \frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{de} = \frac{d\sigma_{\rm PT}}{de} + f_{\rm hadr}(Q,e) \tag{1.1}$$

with e denoting a general event shape variable ($e = t, \rho, C, \dots$) and the subscript

PT referring to perturbative contribution, $\langle e\rangle_{\rm PT} = \int de\, e d\sigma_{\rm PT}/de$. Obviously, the hadronization corrections to the differential distributions have a richer structure then those to the mean values. For instance, non-perturbative scales λ_p parameterizing power corrections to $\langle e\rangle$ are defined by the moment $\int de\, e f_{\rm hadr}(Q,e)$.

Power corrections in (1.1) are associated with hadronization effects in e^+e^--final states and, as a consequence, the magnitude of the scales λ_p and the function $f_{\rm hadr}(Q,e)$ cannot be calculated within perturbative QCD approach. However it was recognized some time ago [3]–[6], that analysis of infrared renormalon ambiguities of perturbative QCD series suggests the value of dimensionless exponents p as well as the dependence of the function $f_{\rm hadr}(Q,e)$ on the large scale Q. Namely, perturbative QCD series generate power corrections of the form (1.1) through IR renormalons contribution but fail to predict uniquely their values — it is only the sum of perturbative and non-perturbative contributions that becomes well defined [7]. To give a meaning to the perturbative series in (1.1) one has to regularize IR renormalon singularities. This can be done in two different ways: one can specify a particular prescription for integrating IR renormalon singularities like principal value prescription [8]. Alternatively, one can avoid IR renormalon ambiguities by introducing an explicit IR cut-off μ on momenta of soft particles in perturbative expressions. In this case, one can either impose a "hard" IR cut-off on momenta of soft particles in the Feynman integrals, $k_\perp > \mu$, [4] or replace QCD coupling constant by a effective IR finite coupling constant which coincides with $\alpha_s(k_\perp)$ at large scale $k_\perp$ and deviates from it at $k_\perp < \mu$ [3, 9]. Following each of these ways, one specifies perturbative (μ-dependent) contribution to (1.1) including perturbatively induced power corrections. Still, there exists a genuine non-perturbative contribution to the event shapes coming from the QCD dynamics at scales below μ. This contribution cannot be determined from the analysis of perturbative QCD series while its magnitude depends on the choice of the IR regularization and, as a consequence, on the IR cut-off μ.

For some hadronic observables like mean values of the event shapes and their differential distributions away from the end-point region, leading non-perturbative power corrections can be parameterized using different IR renormalon inspired phenomenological models [3, 5, 4, 7]. Their predictions agree well with the experimental data and the extracted values of phenomenological non-perturbative parameters exhibit approximate universality. Despite a phenomenological success of these models, it remains still unclear what is the physical meaning of new non-perturbative QCD scales and what is the origin of the universality property within QCD. In the present paper we address these problems using the factorization properties of the event shape distributions established in [10]. We shall argue that non-perturbative power corrections to the thrust, heavy jet mass and C-parameter distributions are described by the universal shape function which is a new non-perturbative QCD distribution measuring the energy flow in the two-jet final states in e^+e^--annihilation.

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The paper is organized as follows. In section 2 we discuss the general properties of power corrections to the event shape distributions in the end-point region. In section 3 we formulate the factorization procedure and define the shape function. In section 4 we show that the differential event shape distributions are given by the convolution of the resummed perturbative cross-sections with universal shape function. Choosing a simple ansatz for this function we compare QCD predictions with the existing data. In section 5 we apply the obtained expressions to calculate the power corrections to the first two moments of the distributions. Concluding remarks are given in section 6.

2. Event shape distributions

In this paper we shall consider three event shape variables: thrust T, heavy jet mass ρ and C-parameter. They are defined in the standard way as [11]

$$T = \max_{\vec{n}_T} \frac{\sum_k |\vec{p}_k \cdot \vec{n}_T|}{\sum_k |\vec{p}_k|}, \qquad \rho = \max\left(\frac{M_R^2}{Q^2}, \frac{M_L^2}{Q^2}\right), \tag{2.1}$$

where M_R^2 and M_L^2 denote the total invariant masses flowing into the right and left hemispheres with respect to the plane orthogonal to the thrust axis $\vec{n}_T$. The C-parameter is given by

$$C = 3(\theta_1\theta_2 + \theta_2\theta_3 + \theta_3\theta_1) \tag{2.2}$$

with θ_j being eigenvalues of space-like part of the energy-momentum tensor $\Theta^{\alpha\beta} = \sum_k p_k^\alpha p_k^\beta / |p_k| / \sum_j |p_j|$.

Introducing the new variable $t = 1 - T$ one notices that thus defined event shapes $e = (t, \rho, C)$ have a number of common features. Lowest order perturbative QCD calculation leads in all three cases to the following expression for the differential distribution for $e > 0$ [11]

$$\frac{d\sigma_{\rm PT}}{de} = \frac{\alpha_s(Q)}{2\pi} A_e(e)\theta(e_{\max} - e) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B_e(e) + \mathcal{O}(\alpha_s^3), \tag{2.3}$$

where A_e and B_e are known coefficient functions and normalization is chosen as $\int de \frac{d\sigma_{\rm PT}}{de} = 1$. Lowest order correction A_e gets contribution only from the three-particle final state which populates the kinematic region $0 \le e \le e_{\max}$ with $t_{\max} = \rho_{\max} = 1/3$ and $C_{\max} = 3/4$. Away from the end-point region, $e \gg \Lambda_{\rm QCD}/Q$, the perturbative expansion (2.3) is well defined and it describes the final states consisting of particles with relative transverse momentum that scales at large center-of-mass energy as $\sim Q$. As e approaches the three-particle upper limit, $e = e_{\max}$, A_t and A_ρ vanish while A_C takes a finite value [11, 12]

$$\begin{aligned} A_t\left(\frac{1}{3}\right) &= A_\rho\left(\frac{1}{3}\right) = 0\,, \\ A_C(C) &= \frac{256}{243}\pi\sqrt{3}C_F\left[1 - \frac{8}{3}\left(C - \frac{3}{4}\right) + \mathcal{O}\big((C - 3/4)^2\big)\right]. \end{aligned} \tag{2.4}$$

For $e = (t, \rho, C) \to \Lambda_{\text{QCD}}/Q$ the final states consist of two narrow jets with invariant mass $M^2_{R,L} \sim \Lambda_{\text{QCD}} Q$. Examining (2.3) one finds that A_e diverges in the end-point region $e \to 0$ as [11]–[14]

$$A_e(e) = \frac{4C_F}{e}\left[\ln\frac{e_0}{e} - \frac{3}{4}\right] + \mathcal{O}(\ln e) \tag{2.5}$$

with $t_0 = \rho_0 = 1$ and $C_0 = 6$. Similar Sudakov-type corrections appear to higher orders, $\alpha_s^N \ln^{2N-n} e/e$ with $n \geq 0$, and need to be resummed [13, 14]. They originate from the effects of collinear splitting of quarks and gluons inside two narrow energetic jets and their interaction with surrounding cloud of soft gluons. The underlying QCD dynamics depends on two infrared scales, Qe and Q^2e, such that $1/Q \ll 1/(Qe^{1/2}) \ll 1/(Qe)$. The smallest scale Qe sets up the typical energy carried by soft gluons, while the scale $Q\sqrt{e}$ defines the transverse momenta of the jets, $k_\perp^2 = Q^2 e$. Applying the standard IR renormalon analysis and examining sensitivity of perturbative expressions with respect to emission of particles on each of these scales, that is soft gluons with energy $\sim Qe$ and collinear particles with the transverse momentum $\sim Q^2 e$, one finds that non-perturbative corrections to the differential distribution appear suppressed by powers of both scales. Then, in the end-point region, $e = \mathcal{O}(\Lambda_{\text{QCD}}/Q)$, we may use the fact that $Qe = \mathcal{O}(\Lambda_{\text{QCD}})$ and expand the differential distribution in powers of larger scale $Q^2 e$. Keeping only the leading term of the expansion one gets

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma}{de} = \sigma_0\left(\alpha_s(Q), \ln e, \frac{1}{Qe}\right) + \mathcal{O}\left(\frac{1}{Q^2 e}\right), \tag{2.6}$$

where σ_0 resums perturbative corrections in $\alpha_s(Q)$ as well as power corrections on the smallest scale Qe

$$\sigma_0 = \frac{d\sigma_{\text{PT}}}{de} + \sum_{k=1}^{\infty} \frac{\lambda_k}{(Qe)^k}\Sigma_k(\alpha_s(Q), \ln e)\,. \tag{2.7}$$

Here, Σ_k are dimensionless perturbative coefficient functions and λ_k are some non-perturbative scales, depending, in general, on the choice of the event shape variable. Using (2.7) we notice that the power corrections have a different form for $e \gg \Lambda_{\text{QCD}}/Q$ and $e \sim \Lambda_{\text{QCD}}/Q$.

For e away from the end-point region, $e \gg \Lambda_{\text{QCD}}/Q$, one may keep in (2.7) only the first term

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma}{de} = \frac{d\sigma_{\text{PT}}}{de} + \frac{\lambda_1}{Qe}\Sigma_1(\alpha_s(Q), \ln e) + \mathcal{O}\left(\frac{1}{(Qe)^2}\right). \tag{2.8}$$

The coefficient function Σ_1 can be found using the well-known property [4, 15] that the *leading* $1/Q$-power correction to the differential distribution (2.8) is generated by a shift of perturbative spectrum, $e \to e - \lambda_1/Q$. This leads to

$$\Sigma_1(\alpha_s(Q), \ln e) = -\frac{d}{d\ln e}\left[\frac{d\sigma_{\rm PT}}{de}\right]. \tag{2.9}$$

Then, it follows from (2.8) that for $e \gg \Lambda_{\rm QCD}/Q$ the leading power corrections to the differential distributions have a rather simple structure: they are parameterized by a single non-perturbative scale λ_1. The same scale determines $1/Q$-power correction to the mean value $\langle e\rangle$. The QCD predictions (2.8) are in a good agreement with the experimental data and the value of λ_1 has been fitted for different shape variables [1]. It is worthwhile to note that in the performed analysis of power corrections to the differential distributions [1] the fitting range of event shape variables was restricted to the region $e \gg \Lambda_{\rm QCD}/Q$. At the same time, as we shall argue below, it is in the region $e \sim \Lambda_{\rm QCD}/Q$ where a novel QCD regime is realized and the structure of hadronization corrections is drastically changed.

For $e \sim \Lambda_{\rm QCD}/Q$ one finds that all terms in (2.7) become equally important and, therefore, need to be resummed to all orders in $1/(Qe)$. The resummation is based on the remarkable factorization properties of the differential distributions. As was shown in [10], the non-perturbative corrections to the leading asymptotic term σ_0 are factorized out into non-perturbative distribution function, the so-called shape function. The general factorized expression for differential distribution looks like [10]

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{de} = \int_0^{eQ} d\varepsilon\, f_e(\varepsilon)\frac{d\sigma_{\rm PT}(e-\varepsilon/Q)}{de} + \mathcal{O}\left(\frac{1}{Q^2 e}\right). \tag{2.10}$$

Its explicit form depends on the choice of the event shape variable $e = (t, \rho, C)$ and will be given in section 4 (see eqs. (4.1), (4.2) and (4.3) below). For $e \gg \Lambda_{\rm QCD}/Q$ one can expand the r.h.s. of (2.10) in powers of $1/Q$ to reproduce the expansion (2.7) with

$$\lambda_n = \int d\varepsilon\, \varepsilon^n f(\varepsilon)\,, \qquad \Sigma_n(\alpha_s(Q), \ln e) = \frac{(-e)^n}{n!}\frac{d^n}{de^n}\left[\frac{d\sigma_{\rm PT}}{de}\right]. \tag{2.11}$$

Expression (2.10) has a simple physical interpretation — non-perturbative corrections increase invariant masses of jets and effectively shift perturbative spectrum towards larger values of the shape variables with the weight given by non-perturbative distribution $f_e(\varepsilon)$.

3. Factorization and shape functions

Factorization relations (2.10) take a simple form for the radiator functions $R(e)$ defined as [13]

$$R(e) = \int_0^e de' \frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{de'} \equiv \langle\theta(e - e(N))\rangle\,. \tag{3.1}$$

Here, $\langle \cdots \rangle$ denotes averaging over all possible final states in e^+e^--annihilation with the weight given by the differential distribution $1/\sigma_{\rm tot} d\sigma/de$ and $e(N)$ denotes the value of the event shape variable e for a given final state $|N\rangle$. Calculating $R(e)$ in perturbation theory one finds

$$R_{\rm PT}(e) = 1 - \frac{\alpha_s(Q)}{2\pi} \int_e^{e_{\rm max}} de' A_e(e') + \mathcal{O}(\alpha_s^2(Q))\,. \tag{3.2}$$

Close to the two-jet region, $e \to 0$, perturbative expressions for $R(e)$ involve Sudakov logs $\alpha_s^N \ln^{2N-n} e$ with $n \geq 0$. In the case of event shapes under consideration, $e = (t, \rho, C)$, these corrections can be systematically resummed to next-to-leading logarithmic (NLL) order and matched into exact two-loop perturbative expressions (3.2) [13, 14]. To this accuracy the weights $e(N)$ can be expressed in terms of the total invariant masses M_R^2 and M_L^2 of two jets flowing into the right and left hemispheres, respectively. Moreover, the t- and C-parameters depend only on the sum of two masses and the corresponding perturbative radiation functions can be expressed to the NLL approximation as [13, 14]

$$\begin{aligned} R_t^{\rm PT}(e) &= \langle \theta(e - t(N)) \rangle_{\rm PT} = \left\langle \theta\left(e - \frac{M_R^2 + M_L^2}{Q^2}\right)\right\rangle_{\rm PT}\,, \\ R_C^{\rm PT}(e) &= \langle \theta(e - C(N)) \rangle_{\rm PT} = \left\langle \theta\left(e - 6\frac{M_R^2 + M_L^2}{Q^2}\right)\right\rangle_{\rm PT} = R_t^{\rm PT}\left(\frac{e}{6}\right)\,, \end{aligned} \tag{3.3}$$

where the subscript PT indicates that the final states in e$^+$e$^-$-annihilation are generating by perturbative branching of outgoing quark and antiquark. The radiator function for the ρ-parameter depends separately on the masses of two jets. Taking into account that perturbative evolution of two jets is independent on each other to the NLL approximation one gets [13]

$$R_\rho^{\rm PT}(e) = \langle \theta(e - \rho(N)) \rangle_{\rm PT} = \left\langle \theta\left(e - \frac{M_R^2}{Q^2}\right)\right\rangle_{\rm PT} \left\langle \theta\left(e - \frac{M_L^2}{Q^2}\right)\right\rangle_{\rm PT}\,. \tag{3.4}$$

The perturbative expressions (3.3) and (3.4) are valid in the two-jet kinematical region $\Lambda_{\rm QCD}/Q \ll e < e_{\rm max}$ except the end-point region $e \sim \Lambda_{\rm QCD}/Q$, in which the energy of emitted soft particles scales as $k_\perp \sim eQ \sim \Lambda_{\rm QCD}$ and perturbation theory is expected to fail.

Calculating the radiator functions $R_e(e)$ one has to combine together perturbative and non-perturbative corrections. In the case of inclusive distributions, like deep inelastic structure functions and Drell-Yan distributions, this can be achieved by applying the factorisation theorems. They allow to separate short-distance dynamics into perturbatively calculable coefficient functions and absorb large-distance corrections into universal non-perturbative distributions. Specific feature of the differential event-shape distributions is that they are not inclusive quantities but rather weighted cross-sections and, as a consequence, the standard methods are not applicable in this case.

It turns out [10] that IR factorization still holds for the leading term σ_0 in the expansion of the event-distributions (2.6) in the end-point region $e \sim \Lambda_{\rm QCD}/Q$. Its origin has a simple physical interpretation. In end-point region, the final state in e^+e^--annihilation consists of two narrow jets surrounding by a cloud of soft gluons. Nonperturbative corrections $\sim 1/(Q^2 e)$ and $\sim 1/(Qe)$ are associated with emission of collinear particles with the transverse momenta $k_\perp^2 \sim Q^2 t$ and soft particles on the energy scale $k_\perp \sim Qe$, respectively. Neglecting power corrections to (2.6) on a larger scale, $\sim 1/(Q^2 e)$, we may restrict analysis to soft particles only. Since soft particle cannot resolve the internal structure of narrow jets of transverse size $k_\perp^2 \sim Q^2 e$, we may effectively replace two jets by a pair of energetic quark and antiquark moving back-to-back with the energy $\sim Q/2$. The internal dynamics of two jets is governed by perturbative branching of quark and antiquark while effects of their interaction with soft gluons can be factorized out into the eikonal phase $W_+ W_-^\dagger$ with W_+ and W_- being the eikonal phases of quark and antiquark, respectively. They are given by Wilson lines $W_\pm = P\exp(i\int_0^\infty ds n_\pm A(sn_\pm))$ in which soft gluon field $A_\mu(x)$ is integrated along the light-like directions $n_\pm$ defined by the momenta of two outgoing jets. In the end-point region, collinear and soft particles provide additive contributions to the shape variables $e = (t, \rho, C)$. As a consequence, the radiator functions are given in all three cases by a convolution of perturbative radiators $R_{\rm PT}$ and the same universal non-perturbative distribution $f(\varepsilon_R, \varepsilon_L)$ describing the energy flow into the right and left hemispheres in the final state, ε_R and ε_L, respectively, created by non-perturbative soft gluon radiation. The non-perturbative distribution $f(\varepsilon_R, \varepsilon_L)$ is defined as follows [10]

$$f(\varepsilon_R, \varepsilon_L) = \sum_N |\langle 0|W_+ W_-^\dagger|N\rangle|^2 \delta(\varepsilon_{_R} - (k_{_R} n_+))\delta(\varepsilon_{_L} - (k_{_L} n_-))\,. \qquad (3.5)$$

Here, sum goes over all possible soft gluon final states $|N\rangle$ with k_R and k_L being the total momentum of soft particles moving into right and left hemispheres, respectively. The quantities $(k_{_R} n_+)$ and $(k_{_L} n_-)$ define the projection of the soft gluon momenta onto the directions of two jets, $n_\pm^\mu = (1, \mathbf{0}_\perp, \pm 1)$, propagating into the same hemisphere.

Finally, the factorized expressions for the radiator function for the t- and C-variables look like

$$R_t(e) = \int_0^{eQ} d\varepsilon\, f_t(\varepsilon)\, R_t^{\rm PT}\left(e - \frac{\varepsilon}{Q}\right), \qquad (3.6)$$

$$R_C(e) = \int_0^{\frac{2}{3\pi}eQ} d\varepsilon\, f_t(\varepsilon)\, R_C^{\rm PT}\left(e - \frac{3\pi}{2}\frac{\varepsilon}{Q}\right) \qquad (3.7)$$

with non-perturbative distribution $f_t(\varepsilon)$ defined as

$$f_t(\varepsilon) = \int d\varepsilon_R \int d\varepsilon_L f(\varepsilon_R, \varepsilon_L)\, \delta(\varepsilon - \varepsilon_R - \varepsilon_L) = \int_0^{\varepsilon} d\varepsilon'\, f(\varepsilon - \varepsilon', \varepsilon')\,. \tag{3.8}$$

In the case of the ρ-variable,

$$R_\rho(e) = \int_0^{eQ} d\varepsilon_R \int_0^{eQ} d\varepsilon_L\, f(\varepsilon_R, \varepsilon_L) R_J^{\rm PT}\left(e - \frac{\varepsilon_R}{Q}\right) R_J^{\rm PT}\left(e - \frac{\varepsilon_L}{Q}\right) \tag{3.9}$$

with $R_\rho^{\rm PT}(e) = [R_J^{\rm PT}(e)]^2$ and $R_J^{\rm PT}(e) = \langle \theta\left(e - M_R^2/Q^2\right)\rangle_{\rm PT}$ being a single jet radiator function. We would like to stress that eqs. (3.6), (3.7) and (3.9) hold in the region $\Lambda_{\rm QCD}^2/Q^2 < e < e_{\max}$. They resum all power corrections of the form $1/(Qe)^n$ and are valid up to corrections $\sim 1/(Q^2 e)$. According to (3.6), (3.7) and (3.9), the power corrections have a different form for ρ and $e = (t, C)$ variables. In the latter case, the radiator function depends on an overall energy flowing into both hemispheres and described by the integrated distribution (3.8).

Nonperturbative corrections to the radiator functions (3.6), (3.7) and (3.9) are governed by the universal shape function $f(\varepsilon_R, \varepsilon_L)$. This function is different from the well-known inclusive QCD distributions and its operator definition was given in [10]. Using (3.5) it is straightforward to show that $f(\varepsilon_R, \varepsilon_L)$ is a symmetric function of its arguments, it does not depend on the center-of-mass energy Q and is normalized as

$$\frac{d}{dQ^2} f(\varepsilon_R, \varepsilon_L) = 0\,, \qquad \int d\varepsilon_R \int d\varepsilon_L\, f(\varepsilon_R, \varepsilon_L) = 1\,, \tag{3.10}$$

where the last relation follows from unitarity of the eikonal phase $W_+ W_-^\dagger$. The matrix element entering (3.5) does not depend on any kinematical scale and, as a consequence, the momenta of soft gluons contributing to (3.5) are not restricted from above. To separate the region of small gluon momenta one has to introduce the factorisation scale μ. Then, the shape function describes the contribution of gluons with $k_\perp < \mu$, while the contribution of gluons with $k_\perp > \mu$, is absorbed into perturbative radiator function $R(e)$. In this way, both non-perturbative shape function and perturbative radiator become μ-dependent while this dependence cancel in their convolution (3.6), (3.7) and (3.9). Since the μ-dependence of radiator function $R_{\rm PT}$ can be calculated perturbatively, the above condition allows to obtain the evolution equations on the non-perturbative distributions [10]. Clearly, there exists an ambiguity in implementing IR cut-off inside perturbative expressions. Different prescriptions correspond to different ways of regularizing IR renormalon singularities and therefore lead to the different expressions for the non-perturbative distributions. In what follows we shall impose a "hard" IR cut-off [10] on gluon momenta inside the perturbative radiator functions entering (3.6), (3.7) and (3.9) as [10]

$$R_{\rm PT}(e) \longrightarrow R_{\rm PT}(e;\mu) = \theta\left(e - \frac{\mu}{Q}\right) R_{\rm PT}^{\rm NLL}(e) + \theta\left(\frac{\mu}{Q} - e\right) R_{\rm PT}^{\rm NLL}\left(\frac{\mu}{Q}\right). \tag{3.11}$$

Throughout the paper we shall substitute $R_{\rm PT}^{\rm NLL}(e)$ by its perturbative expression resummed to the NLL accuracy and matched into two-loop explicit expressions within the modified $\ln R$-matching scheme [13].

Thus defined radiator function (3.11) depends on two scales, $\Lambda_{\rm QCD}$ and IR cut-off μ, that we choose as

$$\Lambda_{\rm QCD} = \mu = 0.25\,{\rm GeV}\,. \tag{3.12}$$

This choice corresponds to the value $\alpha_s(M_Z) = 0.1213$ used for the two-loop perturbative contribution in the $\ln R$-matching scheme. Within the prescription (3.11), the "regularized" perturbative spectrum $d\sigma_{\rm PT}(e;\mu)/de = dR_{\rm PT}(e;\mu)/de$ coincides with the $\ln R$-matched perturbative distribution $dR_{\rm PT}^{\rm NLL}(e)/de$ for $\mu/Q < e < e_{\rm max}$ and it vanishes inside the non-perturbative "window" $0 < e < \mu/Q$. Choosing the value of μ in (3.12) one has to be sure that the end-point of the perturbative distribution, $e = \mu/Q$, belongs to applicability range of the NLL resummed radiator function $R_{\rm PT}^{\rm NLL}(e)$ [13], $2\beta_0\alpha_s(Q^2)\ln e < 1$. Despite the fact that the perturbative spectrum is well defined at $e = \mu/Q$ we do not expect that it provides a reasonable description of the physical distribution in the end-point region. Indeed, it is in this region that non-perturbative power corrections become dominant.

4. Differential distributions

Differentiating the radiator functions (3.6) and (3.7) we obtain the following expressions for the differential t-distribution

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma_t}{de} = Qf(Qe;\mu)R_t^{\rm PT}(0;\mu) + \int_0^{Qe} d\varepsilon\, f_t(\varepsilon;\mu)\frac{d\sigma_t^{\rm PT}(e-\varepsilon/Q;\mu)}{de} \tag{4.1}$$

and C-distribution

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma_C}{de} = \frac{2}{3\pi}Qf\left(\frac{3\pi}{2}Qe;\mu\right)R_C^{\rm PT}(0;\mu) + \int_0^{\frac{2}{3\pi}Qe} d\varepsilon\, f_t(\varepsilon;\mu)\frac{d\sigma_C^{\rm PT}\left(e-\frac{3\pi}{2}\frac{\varepsilon}{Q};\mu\right)}{de}\,. \tag{4.2}$$

Here, we indicated explicitly the dependence of non-perturbative shape function and perturbative distributions on the factorization scale μ. Two terms entering the r.h.s. of (4.1) and (4.2) have the following interpretation. Since the shape function $f_t(\varepsilon)$ rapidly vanishes for large ε, the first term contributes inside the non-perturbative window $0 \le e < \mu/Q$. In this region the emission of perturbative real soft gluons is suppressed due to cut-off imposed on soft gluon momenta $k_\perp > \mu$ and the shape of the distribution is governed entirely by non-perturbative function $f_t(\varepsilon)$. Additional Sudakov factor $R^{\rm PT}(0;\mu)$ takes into account the contribution of virtual soft gluons with $\mu < k_\perp < Q$ and it rapidly vanishes as μ decreases. The second term in (4.1) and (4.2) defines the spectrum inside the perturbative window $\mu/Q < e < e_{\rm max}$. In this region, non-perturbative corrections smear the perturbative spectrum over the interval $\Delta e \sim \Lambda_{\rm QCD}/Q$.

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For the heavy mass distribution one finds

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma_\rho}{de} = Qf_\rho(eQ,eQ;\mu)R_J^{\text{PT}}(0;\mu) + \int_0^{eQ} d\varepsilon\, f_\rho(\varepsilon,eQ;\mu)\frac{d\sigma_J^{\text{PT}}(e-\varepsilon/Q;\mu)}{de}\,, \quad (4.3)$$

where $d\sigma_J^{\text{PT}}/de$ is single jet distribution resummed to the NLL order and defined by the radiator function (3.9), $d\sigma_J^{\text{PT}}/de = dR_J^{\text{PT}}(e)/de$. The heavy mass non-perturbative distribution is given by

$$f_\rho(\varepsilon,eQ) = 2\int_0^{eQ} d\varepsilon'\, f(\varepsilon,\varepsilon')R_J^{\text{PT}}\left(e-\frac{\varepsilon'}{Q}\right). \quad (4.4)$$

Comparing (4.3) with (4.1) and (4.2) we notice that the factorized expressions for the differential t-, C- and ρ-distributions have a similar form but the structure of power corrections is different in the case of the heavy mass. In distinction with (3.8), the heavy mass non-perturbative function f_ρ depends on the shape variable, e, and the center-of-mass energy, Q. This dependence is controlled by perturbative radiator function and has the following interpretation. In the two-jet limit, the invariant mass of each jet is given by the sum of perturbative and non-perturbative contributions, $M_R^2 = M_{R,\text{PT}}^2 + \varepsilon_R Q$ and $M_L^2 = M_{L,\text{PT}}^2 + \varepsilon_L Q$. Perturbative radiation leads to $M_{R,\text{PT}}^2/Q^2 \sim M_{L,\text{PT}}^2/Q^2 \sim \mathcal{O}(\alpha_s(Q))$, while non-perturbative contribution scales as $\varepsilon_R \sim \varepsilon_L \sim \mathcal{O}(\Lambda_{\text{QCD}})$. In contrast with the t-variable, which depends on the sum of both masses and therefore is additive with respect to perturbative and non-perturbative contributions, the ρ-parameter is defined by the largest mass for which the "additivity" property is lost. Namely, comparing invariant masses flowing into two hemispheres one encounters a situation when masses of two perturbative jets are of the same order, $M_{R/L,\text{PT}}^2 = \mathcal{O}(Q^2)$, while their difference is much smaller $|M_{L,\text{PT}}^2 - M_{R,\text{PT}}^2| = \mathcal{O}(Q\Lambda_{\text{QCD}})$.[1] In this case, non-perturbative correction to the difference of the jet masses becomes comparable with the perturbative contribution $|M_{L,\text{PT}}^2 - M_{R,\text{PT}}^2| \sim Q|\varepsilon_L - \varepsilon_R|$, and therefore it can invert the perturbative hierarchy of jet masses, $M_{R,\text{PT}}^2 < M_{L,\text{PT}}^2$, into $M_R^2 > M_L^2$, for instance. This indicates that the power corrections to the ρ-parameter depend not only on the total energy flowing into the both hemispheres as in the case of the thrust variable but also on the *correlations* between ε_L and ε_R. These correlations are induced by the "cross-talk" between particles propagating into different hemispheres in the final state and they should be taken into account by a particular choice of the ansatz for the non-perturbative function $f(\varepsilon_R,\varepsilon_L)$ (see eq. (4.6) below).

According to (4.1), (4.2) and (4.3) the non-perturbative corrections to the t-, C- and ρ-distributions involve two different non-perturbative functions. They are related however to the same universal non-perturbative shape function (3.5) describing

[1] To see that this configuration is not rare it is enough to notice that it corresponds to the vicinity of peak of the perturbative distribution over the difference of the jet masses $|M_L^2 - M_R^2|/Q^2$ [11].

the energy flow into two hemispheres in the final state. We recall, that $f(\varepsilon_R,\varepsilon_L;\mu)$ depends on the cut-off μ imposed on the maximal momenta of soft particles but it is independent on the center-of-mass energy Q. By the definition, $f(\varepsilon_R,\varepsilon_L;\mu)$ distinguishes between particles propagating into right and left hemispheres in the final state and therefore it is not completely inclusive with respect to partonic final states. Namely, it takes into account that quarks and gluons produced at short distances $\sim 1/Q$ and moving into one of the hemispheres will eventually decay at large distances $\sim 1/\Lambda_{\rm QCD}$ and their remnants could flow into opposite hemispheres.[2] This implies that, firstly, in contrast with the well-known inclusive QCD distributions, the shape function $f(\varepsilon_R,\varepsilon_L;\mu)$ is not related to the short distance QCD dynamics and, in particular, its moments can not be related to hadronic matrix elements of *local* composite operators. Indeed, according to the operator definition proposed in [10], the shape functions are defined in terms of the so-called "maximally nonlocal" QCD operators [17, 18]. Secondly, non-inclusive corrections to the shape function describe a "cross-talk" between two hemispheres in the final state leading to correlations between ε_R and ε_L. As a consequence, the shape function is not factorizable into the product of functions depending on the energy flowing into separate hemispheres

$$f(\varepsilon_R,\varepsilon_L) = f_{\rm incl}(\varepsilon_R) f_{\rm incl}(\varepsilon_L) + \delta f_{\rm non\text{-}incl}(\varepsilon_R,\varepsilon_L)\,. \tag{4.5}$$

One should notice that similar property holds for perturbative Sudakov resummed radiator function (3.4). However, one finds that there the factorization holds to the NLL accuracy, eq. (3.4), and non-inclusive corrections first appear at the NNLL level $\sim \alpha_s^2(\alpha_s L)^N$.

In what follows we shall rely on a particular ansatz for the shape function $f(\varepsilon_R,\varepsilon_L)$ which agrees with general properties of non-perturbative QCD distributions and has been used in previous studies of power corrections to the thrust distributions [10]. Namely, one expects that for small values of ε_R and ε_L the shape function should vanish as a power of the energy. Similarly, $f(\varepsilon_R,\varepsilon_L)$ should rapidly vanish as ε_R or ε_L becomes large. Taking into account these properties together with (4.5) one chooses the following expression

$$f(\varepsilon_R,\varepsilon_L) = \frac{\mathcal{N}(a,b)}{\Lambda^2}\left(\frac{\varepsilon_R\varepsilon_L}{\Lambda^2}\right)^{a-1} \exp\left(-\frac{\varepsilon_R^2+\varepsilon_L^2+2b\varepsilon_R\varepsilon_L}{\Lambda^2}\right). \tag{4.6}$$

It depends on two dimensionless parameters a and b and the scale Λ. The factor $\mathcal{N}(a,b)$ is fixed by normalization condition (3.10).

The free parameters, a, b and Λ, have the following meaning. The exponent a determines how fast the shape function vanishes at the origin. The scale Λ sets up the typical energy of soft radiation. The parameter b controls the non-inclusive

[2]Similar effect has been studied using the IR renormalon approach in [16].

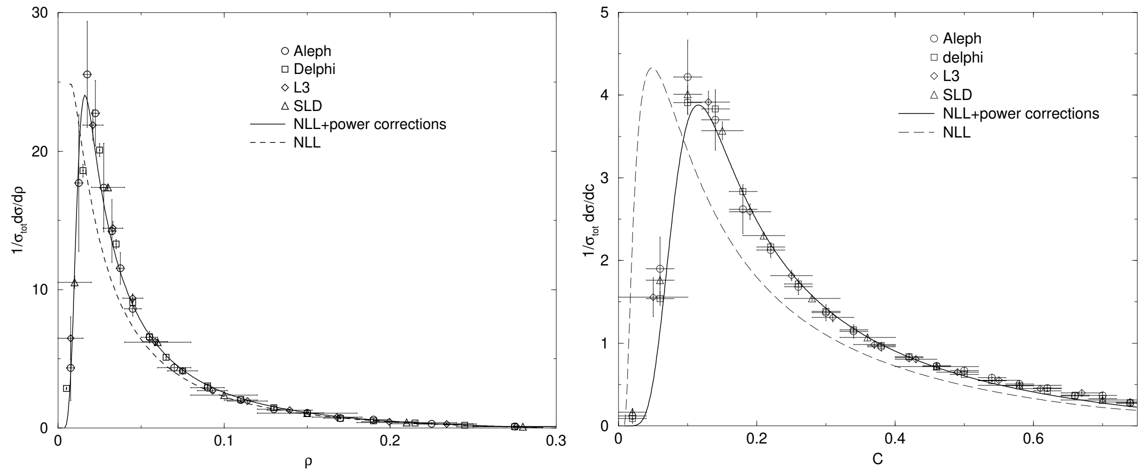
contribution to the shape function and its possible values are restricted as $b > -1$ in order for the shape function (4.6) to be normalizable. Non-inclusive corrections vanish at $b = 0$, $\delta f_{\text{non-incl}} = 0$ in (4.5). For $b \gg 1$, the shape function enhances the regions of the phase space $\varepsilon_R \gg \varepsilon_L$ and $\varepsilon_R \ll \varepsilon_L$, in which most of the energy flows into one of the hemispheres. For $b \to -1$ the energies are of the same order, $\varepsilon_R \sim \varepsilon_L$, and strongly correlated to each other. We expect that non-inclusive corrections to the shape function should be important and the configurations in which energy flows mostly into one of the hemispheres to be suppressed. This suggests that the possible values of the b-parameter should lie within the interval $-1 < b < 0$.

The parameters a, b and Λ depend on the factorization scale μ and are independent on the center-of-mass energy Q as well as the choice of the shape variable $e = (t, \rho, C)$. This allows to fit their values by comparing the event shape distributions, eqs. (4.1), (4.2) and (4.3), with the most precise available experimental data at $Q = M_Z$. Following this procedure we found that the fit to the heavy jet mass distribution is more sensitive to the choice of the parameters (especially to the non-inclusiveness parameter b) then the one to the thrust and the C-parameter. Then, fitting the heavy jet mass distribution at $Q = M_Z$ throughout the interval $0 < \rho < 0.33$ as shown in figure 1a we obtain

$$a = 2\,, \qquad b = -0.4\,, \qquad \Lambda = 0.55\,\text{GeV}\,. \tag{4.7}$$

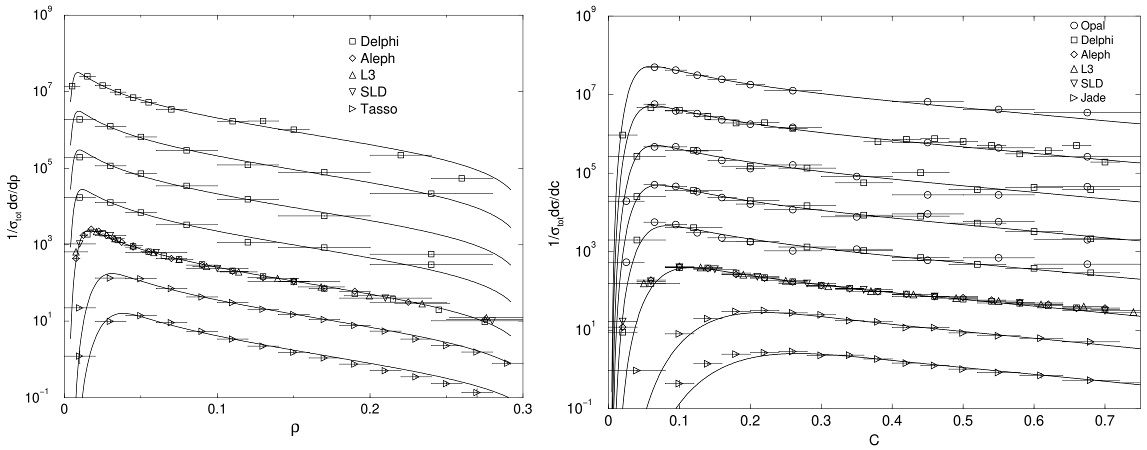
We notice that away from the end-point region, $0.03 < \rho < 0.33$, the experimental data are well described by a simple shift of the perturbative spectrum (2.8) and the corrections in higher powers of $1/(\rho Q)$ are negligible. At the same time, at the vicinity of the peak of the distribution, $0 < \rho < 0.03$, the higher powers of $1/(\rho Q)$ provide equally important contribution and need to be taken into account. Using the values of non-perturbative parameters (4.7) we compare the QCD predictions for the C-parameter distribution at $Q = M_Z$ with and without non-perturbative corrections included as shown in figure 1b. Similar plot for the thrust distribution can be found in [10]. We observe that the differential distributions (4.3) and (4.2) combined with the shape function, eqs. (4.6) and (4.7), correctly describe the data throughout the interval $0 < e < e_{\max}$ including the end-point region $e = \mathcal{O}(\Lambda_{\text{QCD}}/Q)$. In addition, the ρ-parameter distribution turns out to be very sensitive to the choice of the b-parameter. The fact that its value, (4.7), is relatively large indicates that non-inclusive corrections to the shape function (4.5) are important indeed. This result is in accord with our expectations — the parameter b controls the correlations between ε_R and ε_L which, as it was explained before, play an important role for the ρ-parameter.

Having determined the parameters of the shape function, eq. (4.7), at the reference energy scale $Q = M_Z$, we can now apply the factorized expressions for the differential distributions, (4.1), (4.2) and (4.3) with the *same* ansatz for the shape



(a) (b)

Figure 1: Heavy jet mass (a) and C-parameter (b) distributions at $Q = M_Z$ with and without power corrections included.



(a) (b)

Figure 2: Comparison of the QCD predictions for the heavy jet mass (a) and C-parameter (b) distributions with the data at different center-of-mass energies (from bottom to top): $Q/\mathrm{GeV} = 35, 44, 91, 133, 161, 172, 183, 189,$ based on the shape function.

function (4.6) to obtain the QCD predictions at different energy and compare them with the data. The combined plot for the ρ- and C-parameter distributions over the center-of-mass energy interval $35\,\mathrm{GeV} \leq Q \leq 189\,\mathrm{GeV}$ is shown in figure $2a$ and $2b$, respectively. Similar plot for the thrust distribution can be found in [10]. We observe that the theoretical curves reproduce the data over the whole interval of the shape variables including the end-point region.

5. Moments of the event shapes

Recently, the experimental data for the first few moments of various event shape distributions became available [1]. Their analysis indicates a presence of large hadronization corrections whose form deviates from IR renormalon models describing the non-perturbative corrections to the distributions as the shift of perturbative spectrum.

Let us apply the obtained expressions for differential distributions to calculate the first two moments of the t-, C- and ρ-distributions defined as

$$\langle e^n \rangle = \int_0^{e_{\max}} de\, e^n \frac{1}{\sigma_{\rm tot}} \frac{d\sigma}{de}\,, \quad (n = 1\,,\ 2)\,. \tag{5.1}$$

Here, integration goes only over the part of the available phase space, $0 < e < e_{\max}$, corresponding to the three-particle final states, and it does not take into account the contribution of multi-jet final states, $e > e_{\max}$. Quantitative description of hadronization corrections to such final states is not available yet. Putting an upper limit on the value of the shape variable in (5.1) allows us to avoid the latter contribution and to replace the differential distribution $d\sigma/de$ in (5.1) by the obtained expressions (4.1), (4.2) and (4.3) which are valid for $0 < e < e_{\max}$.

Using general expression (2.10) one calculates the mean value of the event shape as

$$\langle e \rangle = \langle e \rangle_{\rm PT} + \frac{\langle \varepsilon \rangle}{Q}\left[1 - e_{\max}\frac{d\sigma_{\rm PT}(e_{\max})}{de}\right] + \mathcal{O}\left(\frac{1}{Q^2}\right), \tag{5.2}$$

where $\langle \cdots \rangle_{\rm PT} = \int_0^{e_{\max}} de\,(\cdots)\, d\sigma_{\rm PT}/de$ denotes averaging with respect to perturbative distribution and the scale $\langle \varepsilon \rangle$ is defined as the first moment of the shape function, $\langle \varepsilon \rangle = \int d\varepsilon\, \varepsilon f(\varepsilon)$. It is important to remember that the factorized expressions for the differential distributions (4.1), (4.2) and (4.3) are valid up to $\mathcal{O}(1/(Q^2 e))$-corrections which may modify the mean value $\langle e \rangle$ by $\mathcal{O}(1/Q^2)$-terms. The additional factor in front of $\langle \varepsilon \rangle/Q$ takes into account that close to the edge of the three-particle phase space, $e \to e_{\max}$, the perturbative distribution (2.3) could take non-zero values

$$e_{\max}\frac{d\sigma_{\rm PT}(e_{\max})}{de} = \frac{\alpha_s(Q)}{2\pi} e_{\max} A_e(e_{\max}) + \mathcal{O}(\alpha_s^2)\,. \tag{5.3}$$

This can be checked using the explicit expressions for perturbative distributions (2.4). We find that $d\sigma_{\rm PT}/de$ vanishes to one-loop order as $e \to e_{\max}$ for the t- and ρ-variables while for the C-parameter it approaches a finite value. Finally, calculating the mean values $\langle \varepsilon \rangle$ with respect to non-perturbative distributions $f_t(\varepsilon)$ and $f_\rho(\varepsilon, eQ)$ defined in (3.8) and (4.4), respectively, we obtain

$$\langle t \rangle = \langle t \rangle_{\rm PT} + \frac{\lambda_1}{Q} + \mathcal{O}\left(\frac{1}{Q^2}\right), \tag{5.4}$$

$$\langle \rho \rangle = \langle \rho \rangle_{\rm PT} + \frac{\lambda_1}{2Q} + \mathcal{O}\left(\frac{1}{Q^2}\right). \tag{5.5}$$

Similarly, for the mean value of the C-parameter we get

$$\langle C\rangle = \langle C\rangle_{\rm PT} + \frac{3\pi}{2}\frac{\lambda_1}{Q}\left[1 - \frac{\alpha_s(Q)}{2\pi}5.73 + \mathcal{O}(\alpha_s^2)\right] + \mathcal{O}\left(\frac{1}{Q^2}\right). \tag{5.6}$$

Here, large perturbative coefficient originates from (2.4) and it reduces a magnitude of the non-perturbative scale λ_1 by 11% at $Q = M_Z$. Relations (5.4) and (5.5) coincide with IR renormalon model predictions [3, 5, 14], while (5.6) differs by perturbative $\alpha_s(Q)$-dependent "boundary" term. Nonperturbative Q-independent scale λ_1 is given by

$$\lambda_1 = \int d\varepsilon_R \int d\varepsilon_L\, (\varepsilon_R + \varepsilon_L) f(\varepsilon_R, \varepsilon_L) = \int d\varepsilon\, \varepsilon f_t(\varepsilon)\,. \tag{5.7}$$

Substituting expression for the shape function, eq. (4.6), one finds $\lambda_1 = \Lambda\,\varphi(a,b)$ with φ given by ${}_2F_1$-hypergeometric series. Using the values of the parameters (4.7) we find

$$\lambda_1 = 1.22\,\text{GeV}\,. \tag{5.8}$$

This value agrees with the results of the recent analysis [21] based on the IR finite QCD coupling constant model [3, 9] and corresponds to the following value of the non-perturbative parameter in that model (with Milan factor taken into account) $\alpha_0 = 0.574$. We would like to recall that the value of λ_1 depends on the factorization scale μ, eq. (3.12), and its μ-dependence is described by QCD evolution equation [10]. Obviously, the value of λ_1, and as a consequence $1/Q$-corrections to the mean values (5.4), (5.5) and (5.6) are less sensitive to the choice of the parameters a, b and Λ as compared with non-perturbative corrections to the corresponding differential distributions.

The comparison of the QCD predictions, (5.4), (5.5) and (5.6), with the data over the energy interval $35\,\text{GeV} \le Q \le 189\,\text{GeV}$ is shown in figure 3. One should notice that (5.4), (5.5) and (5.6) describe the contribution of the two-jet configurations while experimental data take into account all possible final states. A good agreement observed in figure 3 indicates that the contribution to the mean values of the final states with three and more jets as well as $\mathcal{O}(1/(Q^2e))$ subleading corrections to the distributions are subdominant. Indeed, the dominant contribution to the moments (5.1) comes from the vicinity of peak of the differential distribution $e^n d\sigma/de$. Using existing experimental data one can show [19] that for $n = 1$ and $n = 2$ the position of the peak is located in the two-jet kinematical region while for higher n it moves towards larger e for which the integral (5.1) becomes very sensitive to the choice of the upper integration limit $e_{\max}$. We shall use this observation calculating the second moment of the event shape distributions.

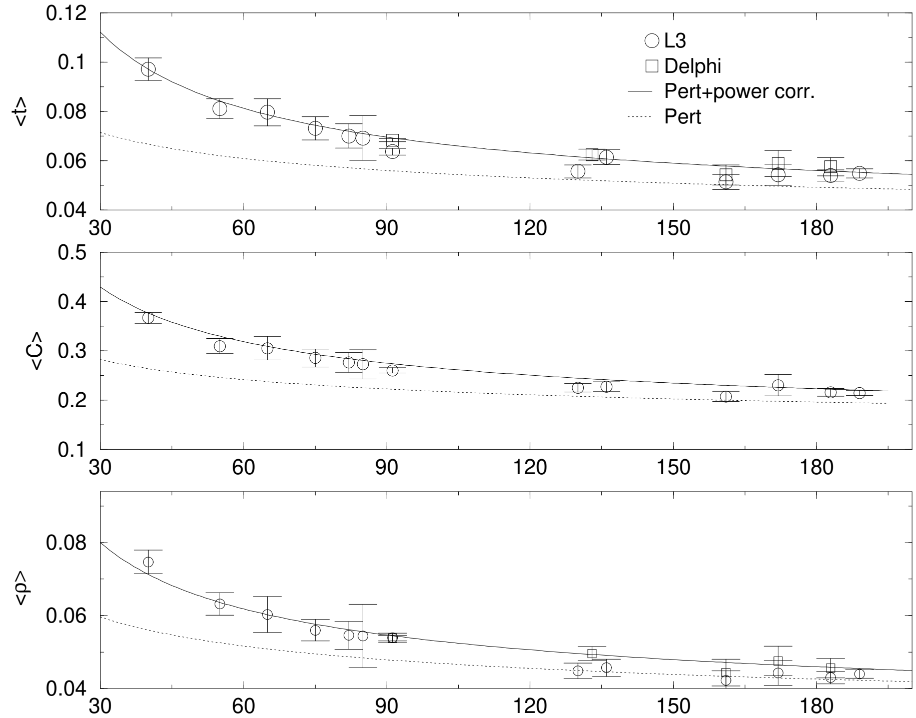


Figure 3: Comparison of the QCD predictions to the mean values $\langle t\rangle$, $\langle\rho\rangle$ and $\langle C\rangle$ with the data. Dotted lines denote $\mathcal{O}(\alpha_s^2)$-perturbative contribution, solid lines take into account power corrections given by eqs. (5.4), (5.5) and (5.6).

Let us apply (5.1) to calculate $\langle e^2\rangle$. Neglecting $1/(Q^2e)$-corrections to the distribution (2.10) we find after some algebra the following general expression:

$$\begin{aligned}\langle e^2\rangle &= \langle e^2\rangle_{\rm PT} + \frac{\langle\varepsilon\rangle}{Q}\left[2\langle e\rangle_{\rm PT} - e_{\rm max}^2\frac{d\sigma_{\rm PT}(e_{\rm max})}{de}\right] + \\ &+ \frac{\langle\varepsilon^2\rangle}{Q^2}\left[1 - e_{\rm max}\frac{d\sigma_{\rm PT}(e_{\rm max})}{de} + \frac{1}{2}e_{\rm max}^2\frac{d^2\sigma_{\rm PT}(e_{\rm max})}{de^2}\right] + \mathcal{O}\left(\frac{1}{Q^3}\right). \qquad (5.9)\end{aligned}$$

Similar to the mean value, (5.2), the boundary terms vanish for the t- and ρ-variables while for the C-parameter they provide a sizeable contribution. Using the explicit expression for the shape functions, (3.8) and (4.4), we calculate the scales $\langle\varepsilon^2\rangle$ and take into account the boundary terms (2.4) to obtain[3]

$$\begin{aligned}\langle t^2\rangle &= \langle t^2\rangle_{\rm PT} + 2\frac{\lambda_1}{Q}\langle t\rangle_{\rm PT} + \frac{\lambda_2}{Q^2}\,, \\ \langle \rho^2\rangle &= \langle \rho^2\rangle_{\rm PT} + \frac{\lambda_1}{Q}\langle \rho\rangle_{\rm PT} + \frac{\lambda_2 + \delta\lambda_2(Q)}{4Q^2}\,, \qquad (5.10) \\ \langle C^2\rangle &= \langle C^2\rangle_{\rm PT} + \frac{3\pi}{2}\frac{\lambda_1}{Q}\left[2\langle C\rangle_{\rm PT} - \frac{\alpha_s(Q)}{2\pi}4.30\right] + \frac{9\pi^2}{4}\frac{\lambda_2}{Q^2}\left[1 - \frac{\alpha_s(Q)}{2\pi}11.46\right].\end{aligned}$$

[3]We are grateful to O. Biebel and S. Kluth for providing us $\mathcal{O}(\alpha_s^2)$-expressions for the moments of the event shapes.

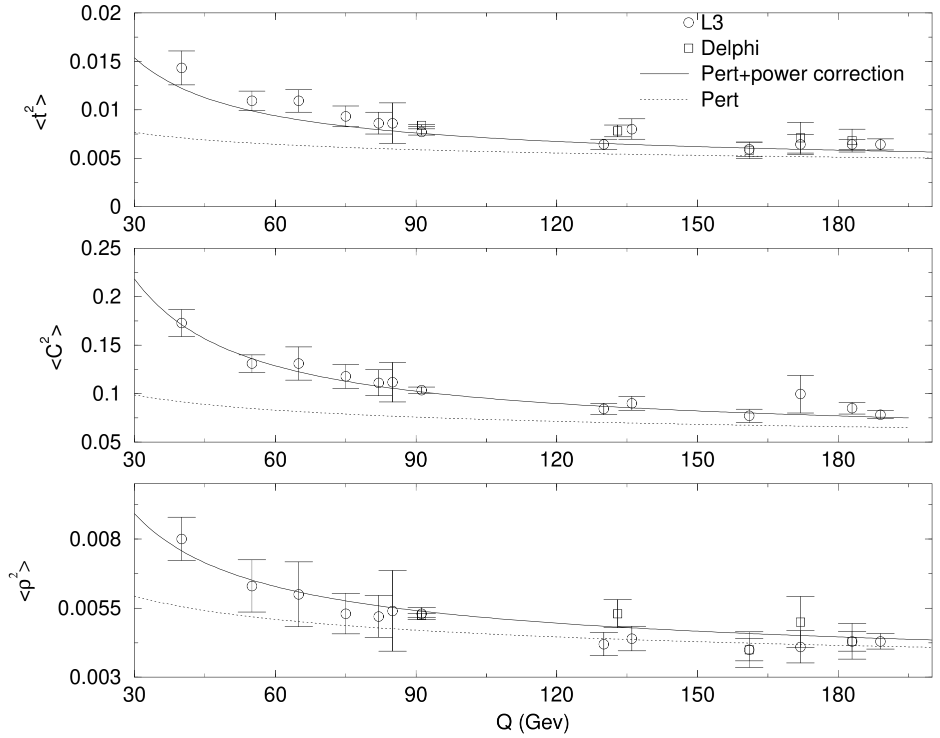


Figure 4: Comparison of the QCD predictions for the second moments $\langle t^2\rangle$, $\langle\rho^2\rangle$ and $\langle C^2\rangle$ with the data. Dotted lines denote $\mathcal{O}(\alpha_s^2)$-perturbative contribution, solid lines take into account power corrections given by eq. (5.10).

Here, the scale λ_1 was defined in (5.7) and new scales λ_2 and $\delta\lambda_2$ are given by

$$\begin{aligned}\lambda_2 &= \langle(\varepsilon_R+\varepsilon_L)^2\rangle\,,\\ \delta\lambda_2(Q) &= \langle(\varepsilon_R-\varepsilon_L)^2\rangle\left\{1+4\int_0^{\rho_{\max}}d\rho'\rho'\left(\frac{d\sigma_J^{\rm PT}}{d\rho'}\right)^2\right\},\end{aligned} \tag{5.11}$$

where average is taken with respect to the shape function $f(\varepsilon_R,\varepsilon_L)$. The Q-dependence of the scale $\delta\lambda_2$ is attributed to perturbative prefactor depending on the single jet distribution, $d\sigma_J^{\rm PT}/d\rho$, defined in (4.3). Its origin was explained in section 4. We find that the value of this factor varies from 2.19 at $Q=10$ Gev to 1.85 at $Q=100$ Gev. It is important to notice that $\langle(\varepsilon_R-\varepsilon_L)^2\rangle$ vanishes if one does not take into account non-inclusive corrections to the shape function (4.5), $\delta f_{\text{non-incl}}=0$. Using (4.6) and (4.7) one gets

$$\lambda_2=1.70\,{\rm GeV}^2\,,\qquad \langle(\varepsilon_R-\varepsilon_L)^2\rangle=0.14\,{\rm GeV}^2\,. \tag{5.12}$$

It follows from (5.10) that the boundary terms generate a sizable perturbative corrections to the second moment of the C-parameter distribution and diminish the magnitude of scales parameterizing $1/Q$-power corrections. Moreover, one finds from (5.4), (5.5), (5.6) and (5.10) that the variance of the distribution, $\langle e^2\rangle-\langle e\rangle^2$, does not receive $1/Q$-power corrections for the t- and ρ-variables while for the C-parameter the boundary terms produce a negative $1/Q$-correction

$$\langle C^2 \rangle - \langle C \rangle^2 = \langle C^2 \rangle_{\rm PT} - \langle C \rangle^2_{\rm PT} - 3.23\, \frac{\lambda_1}{Q} \alpha_s(Q) + \mathcal{O}\left(\frac{1}{Q^2}\right). \qquad (5.13)$$

The comparison of the QCD predictions (5.10) with the experimental data is shown in figure 4. We would like to recall that the obtained expressions for the moments do not take into account the contribution of multi-jet final states configurations and assume a smallness of $1/(Q^2 e)$-corrections to the distributions (2.6). It is interesting to note that the last assumption is supported by the recent analysis of the power corrections to the first two moments of the thrust distribution in the single dressed gluon approximation [20]. The later analysis is complimentary to our studies since it does not resum leading power corrections in the two-jet region and takes into account the contribution coming from the region $e \to e_{\max}$.

6. Conclusions

In this paper we have studied the power corrections to the thrust, t, heavy jet mass, ρ, and C-parameter distributions in the two-jet kinematical region. Our analysis was based on the observation [10] that perturbative and non-perturbative effects can be separated in the differential event shape distributions into calculable Sudakov resummed distribution, $d\sigma_{\rm PT}/de$, and non-perturbative shape function, $f(\varepsilon_R, \varepsilon_L)$, respectively. Each of them depends separately on the factorization scale μ but this dependence cancels in their product. The shape function describes the energy flow into two hemispheres in the final state. It does not depend on the center-of-mass energy Q as well as on the choice of the event shape variable $e = t$, ρ and C.

We demonstrated that away from the end-point region, $e \gg \Lambda_{\rm QCD}/Q$, non-perturbative corrections to the distributions have a simple form (2.8) with the leading $1/Q$-power correction parameterized by a single scale given by the first moment of the shape function. In this region, to which all performed experimental analysis have been restricted so far, our predictions for the thrust and heavy mass distributions and their mean values coincide with those of IR renormalon based models while for the C-parameters we find an additional sizeable perturbative contribution modifying the magnitude of the $1/Q$-power correction (5.6).

In the end-point region, $e \sim \Lambda_{\rm QCD}/Q$, the obtained factorized expressions for the distributions take into account power corrections of the form $1/(Qe)^n$ for arbitrary n. They are controlled by the shape function through (2.11) and are sensitive to the choice of this function. Comparing the QCD predictions with the data we have chosen the simplest ansatz for the shape function (4.6) which is consistent with general properties of non-perturbative distributions and includes non-zero correlations between energy flows into different hemispheres. Examining the dependence of the distributions on the corresponding parameter of the shape function we have observed that these correlations play an important rôle and are not negligible.

In this paper we have restricted our analysis to three different event shape variables: the thrust, heavy-jet mass and C-parameter. The consideration can be extended to other $\mathrm{e^+e^-}$-observables like the energy-energy correlations and jet-broadening B-distributions. In the former case, the power corrections are determined by expectation values of the energy flow operators [10] which in turn are related to the moments of the shape functions (4.6). For the jet-broadening, the non-perturbative scale defining the leading power corrections to the B-distribution in the end-point region, $B \sim \Lambda_{\rm QCD}/Q$, is given by transverse momenta of emitted particles $k_\perp \sim BQ$. The performed analysis can be applied to resum $1/(BQ)^n$-power corrections to the jet-broadening distribution. However, an important difference with respect to the t-distribution is that similar to the ρ- and C-distributions one has to take carefully into account an interplay between perturbative and non-perturbative effects including, in particular, a mismatch between quarks and thrust axis directions [22]. This problem deserves additional studies.

Acknowledgments

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